Marginal Excess Burden Under a Combined Tax System with Linear Commodity and Non-linear Income Taxations

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Abstract

We build up a standard consumption and labour supply choice model with multiple commodities to discuss: (i) the difference of marginal excess burden between cases with and without pre-existing taxes; (ii) the marginal excess burden from a small reform of the non-linear labour income tax system; and (iii) the mutual interactions between commodity and labour supply markets. Firstly, we find that there would be no difference between the two cases unless the whole tax system is linear; and that a big bias arises from a pure income effects and a cross complementarity due to the mutual interactions between commodity and labour markets when we consider the combination of a linear commodity tax and a non-linear labour income tax. Secondly, we extend the work by Blomquist and Simula (2019) and substantiate the importance of the curvature of the non-linear tax function by using the curvature-adjusted compensated elasticity $\tilde{\varepsilon}_n^c$ and income effect parameter $\tilde{\eta}_n$. Thirdly, we focus on the mutual interactions between different markets and we've found the cross complementarity $\partial x_n^{i,c}/\partial z_n$ between the commodity i and labour supply to be another key element in explaining the mutual interactions other than the cross elasticities, $\tilde{\varepsilon}_n^{zj,c}$.

Keywords: marginal excess burden; non-linear taxation; pre-existing taxes
JEL Classification: H20; H21

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1 Introduction

Excess burden or deadweight loss is the efficiency loss associated with taxation and has a long tradition in economics study, dating back as far as to Dupuit (1844). The exact measures of excess burden are various, and the most common concepts are Marshallian surplus and Hicksian variations (see Auerbach, 1985 for an excellent survey). The latter is most commonly used by economists since the former has a serious flaw of path independence. The empirical approximation of excess burden was first pioneered by Harberger with his well-known Harberger triangle in the 1960s (see Harberger, 1962, 1964). Then, the second generation of empirical studies is influenced deeply by Feldstein (1995, 1999), who pointed out that using taxable income elasticity can incorporate various decision margins other than work effort choice.

On the other hand, several recent works highlight the substantial bias of excess burden when ignoring the interactions between different markets. Goulder and Williams (2003) built a general equilibrium model and found out that the interactions between commodity market and labour supply market play an important role in the excess burden, which would cause large bias when ignoring it. Sørensen (2014) then extended their analysis to incorporate more type of taxes, such as capital income taxes, business income tax, etc. As for the excess burden from labour income taxes, previous studies either focus on a linear tax system (Browning, 1987; Auerbach and Hines, 2002), or analyze in the framework with linear taxes by linearizing the non-linear tax system (Feldstein, 1999; Kleven and Kreiner, 2006). However, as Blomquist and Simula (2019) indicates, there is big bias when we use the linearization procedure. Additionally, most previous works have assumed that there eixst no pre-existing taxes and study about welfare effects with pre-existing taxes mainly concentrates on the discussion over optimal green environmental taxes (for example, Bovenberg and Mooij, 1994; Bovenberg and Goulder, 1994).

Inspired by the above mentioned studies, we build up a standard consumption and labour supply choice model with multiple commodities to discuss: (i) the difference of marginal excess burden between cases with and without pre-existing taxes; (ii) the marginal excess burden from a small reform of the non-linear labour income tax system; and (iii) the mutual interactions between commodity and labour supply markets. Firstly, we find that there would be no difference between the two cases unless the whole tax system is linear; and that a big bias arises from a pure income effects and a cross complementarity due to the mutual interactions between commodity and labour markets when we consider the combination of a linear commodity tax and a non-linear labour income tax. Secondly, we extend the work by Blomquist and Simula (2019) and substantiate the importance of the curvature of the non-linear tax function by using the curvature-adjusted compensated elasticity $\tilde{\varepsilon}^c_n$ and income effect parameter $\tilde{\eta}_n$. Thirdly, we focus on the mutual interactions between different markets and we've found the cross complementarity $\partial x^{i,c}_n/\partial z_n$ between the commodity i and labour supply to be another key element in explaining the mutual interactions other than the cross elasticities, $\tilde{\varepsilon}^{zj,c}_n$. In addition, we further discuss two special cases: a piece-wise linear labour income taxation and a HSV tax function to simplify our general results.

The rest of the paper is organized as follows. Section 2 builds up the main model and section 3 gives individuals' behavioral respenses to the two types of reform. Then in section 4 we formulate our main results about the marginal excess burden under cases with pre-existing taxes. In sections 5 and 6 we simplify our results by assuming the labour income tax function to be piece-wise linear and HSV form respectively. At the end of this paper, we concludes and make some remarks on future extensions.

2 The model

2.1 Economic environment

To study the welfare effects of the changes in commodity taxes and non-linear labour income taxes, we follow Mirrlees (1971) and Atkinson and Stiglitz (1976) and build a static consumption and labour supply choice model with multiple goods. There are mass of individuals, who differ with respect to labour productivity n. Labour productivity is distributed over $[\underline{n}, \overline{n}]$ with density function $f(n)$. The preference of individual n is represented by the utility function $u(c_n, x_n, z_n; n)$, where c_n is numeraire, $x_n = \{x^1_n, x^2_n, \cdots, x^S_n\}$ is a bundle of S types of commodities and z_n refers to the before-tax labour income.

Commodity taxes are assumed linear. For any commodity i, the before-tax price is p_i and the tax is t_i, resulting in the post-tax price of commodity i is $q_i = p_i + t_i$. The labour income taxation is denoted by $T(z_n)$, which is non-linear, i.e. $T''(\cdot) \neq 0$. Other types of incomes earned by individual n is denoted by $\overline{I}_n$. Hence, the budget constraint of individual n is $c_n + \sum_{i=1}^{S} q_i x^i_n =$

$z_n - T(z_n) + \bar{I}_n$. Let's define $Q_n = 1 - T'(z_n)$ and $I_n = \bar{I}_n + z_n T'(z_n) - T(z_n)$ as the price of labour income and virtual income respectively. Thus, we can adapt the budget constraint as the standard form, $c_n + \sum_{i=1}^{S} q_i x_n^i = Q_n z_n + I_n$. Overall individual n maximizes his or her utility over $c_n,\ x_n$ and z_n under the aforementioned budget constraint.

2.2 Two-stage method

Following Mirrlees (1976) and Christiansen (1984), we decompose the optimization problem above into two stages. In the first stage, the individual n decides his or her before-tax labour income z_n and aggregate consumption y_n under the income taxation $T(z_n)$. And the budget constraint for stage one is $y_n = Q_n z_n + I_n$. In the second stage, individual n then allocates his or her aggregate consumption y_n to different types of commodities, c_n and $\{x_n^i\}_{i=1}^S$ under the post-tax prices of commodities, $\{q_i\}_{i=1}^S$. Consequently, the budget constraint in the second stage is $c_n + \sum_{i=1}^{S} q_i x_n^i = y_n$. We're going to start with the second-stage problem by using the idea of backward induction.

The second stage. The utility maximization problem for any individual n in the second stage is :

$$v(q, y_n, z_n) \equiv \max_{c_n, x_n} u(c_n, x_n, z_n) \quad \text{s.t.} \quad c_n + \sum_{i=1}^{S} q_i x_n^i = y_n. \tag{1}$$

The optimal allocations and the indirect utility function are denoted by $x_n^i = x_n^i(q, y_n, z_n)$ and $v^n = v(q, y_n, z_n)$ respectively. Likewise, we can write down the expenditure minimization problem given a utility level v^n:

$$e(q, v^n, z_n) \equiv \min_{c_n, x_n} c_n + \sum_{i=1}^{S} q_i x_n^i \quad \text{s.t.} \quad u(c_n, x_n, z_n) \geq v^n. \tag{2}$$

We denote the Hicksian demands and expenditure function by $x_n^{i,c} = x_n^{i,c}(q, v^n, z_n)$ and $e^n = e(q, v^n, z_n)$ respectively.

We omit the details of the optimal conditions and the standard properties such as Roy's identity, Shepard Lemma and Slutsky equations in the consumer theory to save space. The substitution relation between $x_n^{i,c}$, the consumption of commodity i, and z_n the before-tax labour income, however, is highlighted here. Taking partial derivatives of $x_n^{i,c} = x_n^{i,c}(q, v^n, z_n)$ with respect to z_n yields:

$$\frac{\partial x_n^{i,c}}{\partial z_n} = \frac{\partial x_n^i}{\partial z_n} - \frac{\partial x_n^i}{\partial y_n}\frac{u_{z_n}}{u_{c_n}}, \tag{3}$$

where u_{z_n} and u_{c_n} represent the partial derivatives of the utility function respectively. Equation (3) captures how a change in labour supply would affect the demand of the commodity. Like the Slutsky equation, there are substitution effects (the first term) and income effects (the second term). Moreover, $\frac{\partial x_n^{i,c}}{\partial z_n}$ characterizes the complementarity between the consumption of commodity i and the before-tax labour income: when $\frac{\partial x_n^{i,c}}{\partial z_n} > 0$, more labour supply induces more demand of commodity i, indicating that the consumption of commodity i and the before-tax labour income are complementary; when $\frac{\partial x_n^{i,c}}{\partial z_n} < 0$, more labour supply leads to a reduction in consumption, indicating a substitution relation between the consumption of commodity i and the before-tax labour income.

The first stage. We then write down the utility optimization problem for individual n in the first stage:

$$V(q, Q_n, I_n) \equiv \max_{x_n, y_n} v(q, y_n, z_n) \quad \text{s.t.} \quad y_n = Q_n z_n + I_n. \tag{4}$$

The first order condition of problem (4) is $Q_n = -v_{z_n}/v_{y_n} = -u_{z_n}/u_{c_n}$, in which we apply the envelop theorem of problem (1) to obtain the second equation. Then equation (3) can be simplified to:

$$\frac{\partial x_n^{i,c}}{\partial z_n} = \frac{\partial x_n^i}{\partial z_n} + Q_n \frac{\partial x_n^i}{\partial y_n}. \tag{5}$$

The optimal labour income and the indirect utility function are denoted by $z_n = z_n(q, Q_n, I_n)$ and $V^n = V(q, Q_n, I_n)$ respectively. Likewise, we can write down the expenditure minimization problem given a utility level V^n:

$$E(q, Q_n, V^n) \equiv \min_{x_n, y_n} y_n - Q_n z_n \quad \text{s.t.} \quad v(q, y_n, z_n) \geq V^n. \tag{6}$$

We denote the Hicksian demands and expenditure function by $z_n^c = z_n^c(q, Q_n, V^n)$ and $E^n = E(q, Q_n, V^n)$ respectively.

2.3 Elasticities

Firstly, for individual n, we define the compensated demand elasticity of commodity i with respect to a small change in the price of commodity j as

$$\varepsilon_n^{ij,c} = \frac{\partial x_n^{i,c}}{\partial q_j} \frac{q_j}{x_n^i}. \tag{7}$$

Then, we define the compensated elasticity of labour income with respect to a change in labour income tax and the price of commodity j separately as

$$\varepsilon_n^c = \frac{\partial z_n^c}{\partial Q_n} \frac{1 - T'(z_n)}{z_n}, \quad \varepsilon_n^{zj,c} = \frac{\partial z_n^c}{\partial q_j} \frac{q_j}{z_n}. \tag{8}$$

Finally, the income effect parameter of labour supply is defined as

$$\eta_n = \frac{\partial z_n}{\partial I_n}(1 - T'(z_n)). \tag{9}$$

We hereafter assume leisure is not inferior good, and hence the income effect parameter η_n is non positive.

3 Tax reform and behavioral responses

The behavioral responses to price changes are well-known as the substitution effects and income effects. In this section, we focus on how individuals react to a rise in commodity taxes as well as in non-linear labour income tax via substitution and income effects and how these effects vary in a non-linear tax system.

3.1 Behavioral responses to changes in commodity taxes

We now consider a small change in the post-tax price of commodity j, namely $dq_j \approx 0$. Firstly, let's analyze the effects on labour supply. Recall that $z_n = z_n(q, Q_n, I_n)$. It shows that q_j has a direct effect on labour income, i.e. $\partial z_n / \partial q_j$. In the meanwhile, the change in labour income induces changes in the price Q_n and wealth I_n due to the non-linearity of the labour income taxation, then labour income is affected again and repeat ad infinitum. By taking total derivatives of $Q_n =$

$1 - T'(z_n)$ and $I_n = \overline{I}_n + z_n T'(z_n) - T(z_n)$ with respect to q_j respectively, we have the effects of an increase in q_j on the price of labour income and the virtual wealth:

$$\frac{dQ_n}{dq_j} = -T''(z_n)\frac{dz_n}{dq_j}, \tag{10}$$

$$\frac{dI_n}{dq_j} = z_n T''(z_n)\frac{dz_n}{dq_j}. \tag{11}$$

Then, by taking total derivatives of $z_n = z_n(q, Q_n, I_n)$ with respect to q_j and rearrange the terms, we have:

$$\begin{aligned}\frac{dz_n}{dq_j} =& \frac{1 - T'(z_n)}{1 - T'(z_n) + z_n T''(z_n)\varepsilon_n^c} \cdot \frac{\partial z_n}{\partial q_j} \\ =& \underbrace{\frac{1 - T'(z_n)}{1 - T'(z_n) + z_n T''(z_n)\varepsilon_n^c}}_{\text{Curvature Adjustment}} \cdot \left(\frac{\partial z_n^c}{\partial q_j} - \frac{\partial z_n}{\partial I_n} x_n^j\right).\end{aligned} \tag{12}$$

It's evident that the curvature $T''(z_n)$ captures the aforementioned repeated effects and plays an important role in making a difference in behavioral effects via curvature adjustment. Likewise, we have the effects of an increase in q_j on the compensated labour income z_n^c:

$$\frac{dz_n^c}{dq_j} = \frac{1 - T'(z_n)}{1 - T'(z_n) + z_n T''(z_n)\varepsilon_n^c} \cdot \frac{\partial z_n^c}{\partial q_j}. \tag{13}$$

Next, we analyze the effects on the consumption demand. In the standard consumer theory, a price change influences the consumption demand through substitution effect and income effect. In the leisure choice model, however, a price change affects the demand by changing the labour income other than via substitution and income effects. To have a clearer insight, we take total derivatives of $x_n^i = x_n^i(q, y_n, z_n)$ with respect to q_j, and we obtain:

$$\frac{dx_n^i}{dq_j} = \underbrace{\frac{\partial x_n^{i,c}}{\partial q_j} - \frac{\partial x_n^i}{\partial y_n} x_n^j}_{\text{Substitution and Income Effects}} + \underbrace{\frac{\partial x_n^{i,c}}{\partial z_n}\frac{dz_n}{dq_j}}_{\text{Cross Complementarity}}. \tag{14}$$

Obviously, the first and second terms are substitution and income effects respectively, and the third term is the cross complementarity that demonstrates that a price change of commodity j also affects the consumption demand $x_n^{i,c}$ via the complementarity between the consumption and the labour income, i.e. $\partial x_n^{i,c}/\partial z_n$. Likewise, we have the effects of an increase in q_j on the compensated consumption demand $x_n^{i,c}$:

$$\frac{dx_n^{i,c}}{dq_j} = \frac{\partial x_n^{i,c}}{\partial q_j} + \frac{\partial x_n^{i,c}}{\partial z_n}\frac{dz_n}{dq_j}. \tag{15}$$

3.2 Behavioral responses to changes in non-linear labour income taxes

Changes in a linear tax system are straightforward since only the intercept or the marginal tax rates can be varied. However, for a non-linear tax, there are numerous ways to vary the tax system: the intercept, the slope and even the tax system itself can be changed. Closely following Gerritsen (2016), we consider a *reform function* $\tau(\cdot)$, which is assumed twice defferentiable, and a *reform parameter* κ. Then the post-reform labour income tax system for individual n is: $T(z_n, \kappa) = T(z_n) + \kappa\tau(z_n)$. The reform function, $\tau(z)$, is determined by whatever forms of reform we would like to study; and the reform parameter, κ, takes on arbitrary values and measures the size of the tax reform. We assume that z_n is defferentiable in κ and focus on a small change in κ around $\kappa = 0$ below.

Before our analysis on individuals' behavioral responses, we firstly show how the tax reform changes the price of labour income Q_n and the virtual income I_n. Taking total derivatives of the post-reform price $Q_n = 1 - T'(z_n) - \kappa\tau'(z_n)$ and virtual income $I_n = \bar{I}_n + z_n[T'(z_n) + \kappa\tau'(z_n)] - [T(z_n) + \kappa\tau(z_n)]$ with respect to κ respectively, we obtain:

$$\frac{dQ_n}{d\kappa} = -\tau'(z_n) - T''(z_n)\frac{dz_n}{d\kappa}, \tag{16}$$

$$\frac{dI_n}{d\kappa} = -\tau(z_n) - z_n\frac{dQ_n}{d\kappa}. \tag{17}$$

For equation (16), the change in price is opposite to the change in marginal tax rate since the price equals one minus the marginal tax rate by definition. As for equation (17), the first term illustrates that the increment of tax burden directly reduces the labour income by $\tau(z)$, and the second term illustrates that the real wealth increases by $z_n \cdot (-dQ_n)$ when there is a reduction of $-dQ_n$ in the price.

Now we analyze the effects of tax reforms on labour income. By taking total derivatives of

$z_n = z_n(q, Q_n, I_n)$ with respect to κ, and rearranging terms, we have the following equation:

$$\frac{dz_n}{d\kappa} = -\frac{z_n}{1-T'(z_n)}\left[\underbrace{\frac{(1-T'(z_n))\varepsilon_n^c}{1-T'(z_n)+z_nT''(z_n)\varepsilon_n^c}\cdot\tau'(z_n)}_{\text{Curvature Adjusted Substitution Effect}} + \underbrace{\frac{(1-T'(z_n))\eta_n}{1-T'(z_n)+z_nT''(z_n)\varepsilon_n^c}\cdot\frac{\tau(z_n)}{z_n}}_{\text{Curvature Adjusted Income Effect}}\right]. \quad (18)$$

Equation (18) shows that as in the linear tax case, changes in κ affects the labour income via two channels: substitution and income effects. However, the distinction here is that the curvature of the tax system $T''(z_n)$ plays an significant role and captures a repeated effects thanks to the mutual interaction between $Q_n,\ I_n$ and z_n.

Further, in the light of the concepts of elasticity in the linear case, we define the elasticities under the non-linear tax system. Let's denote the non-linear compensated elasticity and the income elasticity by $\tilde{\varepsilon}_n^c$ and $\tilde{\eta}_n$ respectively, and they takes on the following forms [1]:

$$\tilde{\varepsilon}_n^c = \frac{(1-T'(z_n))\varepsilon_n^c}{1-T'(z_n)+z_nT''(z_n)\varepsilon_n^c}, \quad (19)$$

$$\tilde{\eta}_n = \frac{(1-T'(z_n))\eta_n}{1-T'(z_n)+z_nT''(z_n)\varepsilon_n^c}. \quad (20)$$

Armed with concepts of elasticity, we then simplify equation (18) to the elasticity form:

$$\frac{dz_n}{d\kappa} = -\frac{z_n}{1-T'(z_n)}\left[\tilde{\varepsilon}_n^c\tau'(z) + \tilde{\eta}_n\frac{\tau(z_n)}{z_n}\right]. \quad (21)$$

[1] According to Gerritsen (2016), a reform of labour income at income level z raises the marginal tax rates by $\tau'(z)d\kappa$ and the absolute tax burden by $\tau(z)d\kappa$. Then, the compensated elasticity $\tilde{\varepsilon}^c$ can be defined as the relative change in the taxable income due to the relative change in the marginal tax rate when holding the absolute tax burden constant. This means:

$$\tilde{\varepsilon}_n^c \equiv \frac{dz_n}{z_n}\Big/\frac{-\tau'(z_n)d\kappa}{1-T'(z_n)}\bigg|_{\tau(z_n)=0} = \frac{1-T'(z_n)}{z_n}\frac{dz_n}{-\tau'(z_n)d\kappa}\bigg|_{\tau(z_n)=0}.$$

Similarly, the uncompensated elasticity $\tilde{\varepsilon}_n^u$ is defined as the relative change in the taxable income due to the relative change in marginal tax rates when the increase of marginal tax rate and average tax rate are equal, which means:

$$\tilde{\varepsilon}_n^u \equiv \frac{dz_n}{z_n}\Big/\frac{-\tau'(z_n)d\kappa}{1-T'(z_n)}\bigg|_{\tau(z_n)/z_n=\tau'(z_n)} = \frac{1-T'(z_n)}{z_n}\frac{dz_n}{-\tau'(z_n)d\kappa}\bigg|_{\tau(z_n)/z_n=\tau'(z_n)}.$$

Then the income effect is the difference between these two elasticities. And such definitions correspond to our derivations.

Likewise, we have the effects of an increase in κ on the compensated labour income z_n^c:

$$\frac{dz_n^c}{d\kappa} = -\frac{z_n}{1-T'(z_n)} \frac{(1-T'(z_n))\varepsilon_n^c}{1-T'(z_n)+z_n T''(z_n)\varepsilon_n^c} \cdot \tau'(z_n) = -\frac{z_n}{1-T'(z_n)} \tilde{\varepsilon}_n^c \tau'(z). \tag{22}$$

Next, we analyze the effects on consumption demand. Similarly, by taking total derivatives of $x_n^i = x_n^i(q, y_n, z_n)$ and $x_n^{i,c} = x_n^{i,c}(q, v^n, z_n)$ with respect to κ, we obtain:

$$\frac{dx_n^i}{d\kappa} = \frac{\partial x_n^{i,c}}{\partial z_n} \cdot \frac{dz_n}{d\kappa} - \frac{\partial x_n^i}{\partial y_n} \tau(z_n), \tag{23}$$

$$\frac{dx_n^{i,c}}{d\kappa} = \frac{\partial x_n^{i,c}}{\partial z_n} \cdot \frac{dz_n}{d\kappa}. \tag{24}$$

4 Marginal excess burden

In this section, we compare two types of concepts of excess burden: in the first situation without pre-existing taxes, the initial equilibrium is Pareto optimal; in the second situation with pre-existing taxes, the initial equilibrium is not Pareto optimal. We find that there doesn't exist bias between these two situations when the whole tax system is linear; and that bias arises from a pure income effect and a cross complementarity when the labour income taxation is non-linear. Besides, we highlight the importance of the curvature of the non-linear tax function and the interactions between the consumption and labour supply markets.

4.1 Without pre-existing taxes

Adapting the equivalent variation measure, the excess burden from taxation is the difference between the amount that the individual would give up to get rid of all taxes and the tax revenue collected by the government. Suppose the tax system is $\{t_i\}_{i=1}^S$ and $T(z_n, \kappa)$, and denote the before-tax parameters by a hat. The equivalent variation for individual n is:

$$\begin{aligned} EV(n) &= (E(q, Q_n, V(q, Q_n, I_n)) - I_n) - (E(\hat{q}, \hat{Q}_n, V(q, Q_n, I_n)) - \hat{I}_n) \\ &= \hat{I}_n - E(\hat{q}, \hat{Q}_n, V(q, Q_n, I_n)), \end{aligned} \tag{25}$$

where the before-tax prices satisfy $\hat{q} = p$ and $\hat{Q}_n = 1$. Then for individual n, the total excess burden is:

$$EB(n) = EV(n) - \sum_{i=1}^{S} t_i x_n^{i,c} - T(z_n^c, \kappa). \tag{26}$$

From equation (26) it then follows that the marginal excess burden from an increase in commodity or labour income tax is:

$$MEB(n) \equiv \frac{dEB(n)}{d\theta}, \quad \theta = t_1, t_2, \cdots, t_S, \kappa. \tag{27}$$

Then we have the marginal excess burden from a rise in commodity and labour income tax rates, and formulate the results in the following proposition.

Proposition 1 (Marginal excess burden without pre-existing taxes)**.** *The marginal excess burden from a small rise in commodity tax ($dt_j \approx 0$) can be written as*

$$MEB_N^C(n) = -\sum_{i=1}^{S} t_i \varepsilon_n^{ij,c} \frac{x_n^i}{q_j} - \left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \tilde{\varepsilon}_n^{zj,c} \frac{z_n}{q_j}, \tag{28}$$

and the marginal excess burden from a small rise in labour income tax ($d\kappa \approx 0$) can be written as

$$MEB_N^L(n) = -\left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \cdot \tilde{\varepsilon}_n^c z_n \cdot \frac{\tau'(z_n)}{1 - T'(z_n)}, \tag{29}$$

where $\tilde{\varepsilon}_n^{zj,c} = \frac{1-T'(z_n)}{1-T'(z_n)+z_n\varepsilon_n^c T''(z_n)} \varepsilon_n^{zj,c}$ and $\tilde{\varepsilon}_n^c = \frac{1-T'(z_n)}{1-T'(z_n)+z_n\varepsilon_n^c T''(z_n)} \varepsilon_n^c$ are curvature-adjusted compensated elasticities.

From equations (28) and (29), three key points should be highlighted. Firstly, the efficiency loss from an increase in commodity or labour income tax stems from the shrinking of the consumption as well as the labour income tax base. Secondly, the efficiency loss can be decomposed into substitution effects, captured by compensated elasticities and the cross-complementarity effects, captured by the complementarity between consumption and labour income, $\partial x_n^{i,c}/\partial z_n$. The latter plays a significant role as well. When consumption of commodity i and the labour income are substitutes, namely $\partial x_n^{i,c}/\partial z_n < 0$, a rise in tax rate of commodity i creates less efficienct loss than a rise in tax rate of commodity j which is the complementarity of labour income. In other words, in order to minimize the excess burden, the substitutes of labour supply should be levied higher tax while the complementary goods of labour supply should not be taxed heavily. This exactly corresponds to the optimal taxation rule in Corlett and Hague (1953). Thirdly, the sufficient statistics for the marginal excess burden are curvature-adjusted compensated elasticities thanks to the significance of the curvature of the non-linear tax system. Under a non-linear tax system, labour income z_n interacts with its price Q_n and virtual income level I_n mutually and such interactions repeat ad infinitum. Such repeated mutual interactions are captured by the curvature of the tax system $T''(\cdot)$, and disappear when the non-linear tax system degrades into a linear one.

4.2 With pre-existing taxes

When there are pre-existing taxes already, the initial equilibrium is not Pareto optimal, indicating that $\hat{q} \neq p$ and $\hat{Q}_n \neq 1$. Suppose the tax system before the reform is $\{t_i\}_{i=1}^S$ and $T(z)$, and the tax system becomes $\{t_i'\}_{i=1}^S$ and $T(z)$, where $t_j' = t_j + \Delta t_j$ and $t_i' = t_i$ for all $i \neq j$ after a commodity tax reform, and the tax system becomes $\{t_i\}_{i=1}^S$ and $T(z,\kappa) = T(z) + \kappa\tau(z)$ after a labour income tax reform. Following Auerbach (1985), the excess burden from a tax reform is the difference between the amount that an individual would give up to get rid of all the tax reforms and the increase in the tax revenue collected by the government. Then we define the excess burden from a small reform in the commodity tax as:

$$\begin{aligned} EB^C(n) \equiv & EV(n) - \left[\sum_{i=1}^S t_i' x_n^{i,c}(q, v^n, z_n) - \sum_{i=1}^S t_i x_n^{i,c}(\hat{q}, v^n, \hat{z}_n)\right] \\ & - \left[T(z_n^c(q, Q_n, V^n)) - T(z_n^c(\hat{q}, \hat{Q}_n, V^n))\right]. \end{aligned} \tag{30}$$

And the excess burden form a small reform in the labour income tax is defined as:

$$\begin{aligned} EB^L(n) \equiv & EV(n) - \left[\sum_{i=1}^S t_i x_n^{i,c}(q, v^n, z_n) - \sum_{i=1}^S t_i x_n^{i,c}(\hat{q}, v^n, \hat{z}_n)\right] \\ & - \left[T(z_n^c(q, Q_n, V^n)) + \kappa\tau(z_n^c(q, Q_n, V^n)) - T(z_n^c(\hat{q}, \hat{Q}_n, V^n))\right]. \end{aligned} \tag{31}$$

In both equations (30) and (31), both $v^n = v(q, y_n, z_n)$ and $V^n = V(q, Q_n, I_n)$ refer to post-reform welfare level of individual n. Taking total derivative of equation (30) w.r.t. Δt_j around $\Delta t_j = 0$ and taking total derivative of equation (31) w.r.t. κ around $\kappa = 0$ allow us to formulate the following proposition.

Proposition 2 (Marginal excess burden with pre-existing taxes)**.** *The marginal excess burden from a small reform in commodity tax ($d\Delta t_j \approx 0$) can be written as*

$$MEB^C(n) = \underbrace{-\sum_{i=1}^{S} t_i \varepsilon_n^{ij,c} \frac{x_n^i}{q_j} - \left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n}\right) \tilde{\varepsilon}_n^{zj,c} \frac{z_n}{q_j}}_{MEB_N^C(n)}$$
$$+ \overbrace{\underbrace{T'(z_n)(\tilde{\eta}_n - \eta_n) s_j \frac{z_n}{q_j}}_{\text{Labour Supply}} + \underbrace{\sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \tilde{\eta}_n s_j \frac{z_n}{q_j}}_{\text{Cross Complementarity}}}^{\text{Bias: Income Effects}} \quad (32)$$

and the marginal excess burden from a small reform in labour income tax ($d\kappa \approx 0$) can be written as

$$MEB^L(n) = \underbrace{-\frac{\left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n}\right)}{1 - T'(z_n)} \cdot \tilde{\varepsilon}_n^c z_n \tau'(z_n)}_{MEB_N^L(n)}$$
$$+ \overbrace{\underbrace{\frac{T'(z_n)}{1 - T'(z_n)} (\tilde{\eta}_n - \eta_n) \tau(z_n)}_{\text{Labour Supply}} + \underbrace{\frac{\sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n}}{1 - T'(z_n)} \tilde{\eta}_n \tau(z_n)}_{\text{Cross Complementarity}}}^{\text{Bias: Income Effects}}, \quad (33)$$

where $\tilde{\varepsilon}_n^{zj,c} = \frac{1-T'(z_n)}{1-T'(z_n)+z_n\varepsilon_n^c T''(z_n)} \varepsilon_n^{zj,c}$, $\tilde{\varepsilon}_n^c = \frac{1-T'(z_n)}{1-T'(z_n)+z_n\varepsilon_n^c T''(z_n)} \varepsilon_n^c$ and $\tilde{\eta}_n = \frac{1-T'(z_n)}{1-T'(z_n)+z_n\varepsilon_n^c T''(z_n)} \eta_n$ are curvature-adjusted elasticities, and $s_j = \frac{q_j x_n^j}{(1-T'(z_n))z_n}$ refers to the shares of the consumption of commodity j in the after-tax labour income.

Equations (32) and (33) are closely related to the marginal excess burden equations in the case without pre-existing taxes (see equations (28) and (29)), but they differs with a significant bias: income effects, characterized by $\tilde{\eta}_n$ and η_n. There are two sources of income effects induced by the tax reform (either the commodity price change z_n/q_j or the labour income tax reform $\tau(z)$): (i) pure income effects of labour supply; (ii) income effects of cross complementarity between consumption and labour supply.

(i) Pure income effects of labour supply. Why do there exist income effects? It's the non-linear tax system that induces income effects. Recall that the pre and post-reform compensated labour incomes: $z_n^c(\hat{q}, \hat{Q}_n, V(q, Q_n, I_n))$ and $z_n^c(q, Q_n, V(q, Q_n, I_n))$. Under a non-linear tax system, when prices alter, the virtual wealth I_n is affected and in turn affects the labour income z_n via changing the indirect utility function. Then, notice that the post-reform labour income actually equals to $z_n(q, Q_n, I_n)$ by using the dual property. Hence, the curvature-adjusted income effect is generated by the post-reform labour income, while the pre-reform labour income generates the standard income effect. Under a linear tax system, the price Q_n and wealth $I_n = \overline{I}_n$ are never affected by the tax reform, resulting in no income effects. This also explains why there doesn't exist pure income effects of consumption demand.

Another importance of the non-linearity is the progressivity of the tax system affects the sign and size of the bias. Remember the assumption that $\varepsilon^c(n)$ is non negative and $\eta(n)$ is non positive. Thus, when it's a progressive tax system, i.e. $T''(z) > 0$, the curvature-adjusted income effect $\tilde{\eta}_n$ is greater than η_n, resulting in a positive bias; when the tax system is regressive, i.e. $T''(z) < 0$, the curvature-adjusted income effect $\tilde{\eta}_n$ is less than η_n, resulting in a negative bias. And the more progressive or regressive, the higher the absolute value of the bias is.

(ii) Income effects of cross complementarity. Another source of income effects is the complementarity between the consumption demand and the labour income. This term arises obviously because of the mutual interaction between consumption and labour supply and plays an important role in the sign and the size of the bias from cross complementarity. Complements, namely commodities with $\partial x_n^{i,c}/\partial z_n > 0$ tend to augment excess burden, while substitutes, namely commodities with $\partial x_n^{i,c}/\partial z_n < 0$ tend to lessen efficiency loss.

4.3 Marginal excess burden for the population

Up to now, we have defined and derived the marginal excess burden fora single consumer. In order to obtain the overall efficiency loss for the population, we simply integrate the marginal excess burden for a single individual over the population. We regard all individuals equally important in welfare, and then obtain the aggregate excess burden in the following proposition.

Proposition 3 (Aggregate marginal excess burden)**.** *The aggregate marginal excess burden from a small* *commodity tax reform is:*

$$AEB^C = \int_{\underline{n}}^{\overline{n}} \left\{ -\sum_{i=1}^{S} t_i \varepsilon_n^{ij,c} \frac{x_n^i}{q_j} - \left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \tilde{\varepsilon}_n^{zj,c} \frac{z_n}{q_j} \right.$$
$$\left. + T'(z_n)(\tilde{\eta}_n - \eta_n) s_j \frac{z_n}{q_j} + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \eta_n s_j \frac{z_n}{q_j} \right\} f(n) dn, \quad (34)$$

and the aggregate marginal excess burden from a small labour income tax reform is:

$$AEB^L = \int_{\underline{n}}^{\overline{n}} \left\{ -\frac{\left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right)}{1 - T'(z_n)} \cdot \tilde{\varepsilon}_n^c z_n \tau'(z_n) \right.$$
$$\left. + \frac{T'(z_n)}{1 - T'(z_n)} (\tilde{\eta}_n - \eta_n) \tau(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \tilde{\eta}_n \tau(z_n) \right\} f(n) dn. \quad (35)$$

In principle, matters are far more complicated when we wish to aggregate for all individuals because aggregation relies heavily on the wealth distribution, as Auerbach (1985) has discussed. However, simple integration will suffice to show the bias between cases with and without pre-existing taxes.

5 Marginal excess burden and piece-wise linear tax function

In this and next sections, we omit the commodity taxes temporarily and focus our attention on the non-linearity of the labour income taxation. One way to represent the non-linearity of the tax schedule is to assume it as a piece-wise linear tax function. For simplicity, we consider a two-bracket case. Suppose the kink point of taxable income is z^*, and the marginal tax rates of the pre-reform taxation are defined as

$$T'(z) = \begin{cases} t_1, & \text{if } z \leq z^*; \\ t_2, & \text{if } z > z^*. \end{cases} \quad (36)$$

Additionally, we assume the tax reform $\tau(z)$ is also piece-wise linear and its marginal tax rates within the two brackets are τ_1 and τ_2 respectively.

One advantage of using piece-wise linear tax function is its tractability and simplicity for individuals within the brackets since their compensated elasticity $\tilde{\varepsilon}^c_n$ and income effect parameter $\tilde{\eta}_n$ just degrade to the linear ones, i.e. ε^c_n and η_n and the pure income effects of labour supply disappears as well. Thus, the welfare loss for these consumers in proposition (2) simplifies to

$$MEB^L(n) = \frac{T'(z)}{1-T'(z)} \cdot \varepsilon^c(n) z_n \tau'(z_n) = \begin{cases} \frac{t_1}{1-t_1} \cdot \varepsilon^c_n z_n \tau_1, & \text{if } z < z^*; \\ \frac{t_2}{1-t_2} \cdot \varepsilon^c_n z_n \tau_2, & \text{if } z > z^*. \end{cases} \tag{37}$$

However, it seems much more complicated to analyze behavioral responses of individuals at the kink points. One of the solutions is to regard the piece-wise linear tax system as a special continuous function, of whose the curvature is always zero but infinity at some certain kink points. This indicates that for those at the kink point, their compensated elasticity $\tilde{\varepsilon}^c_n$ and income effect $\tilde{\eta}_n$ becomes zero due to $T''(z) \to +\infty$. Thus, the welfare loss for them in (2) simplifies to

$$MEB^L(n) = -\frac{T'(z)}{1-T'(z)} \cdot \eta_n \tau(z) = -\frac{t_1}{1-t_1} \cdot \eta_n \tau(z^*). \tag{38}$$

Assume the distribution of taxable labour income is $h(z)$, and the measure of individuals bunching at the kink point is $h(z^*)$. Then aggregating different types of individuals can simplify equation (35) to

$$AEB^L = \overline{\varepsilon^c_1} \cdot \frac{t_1}{1-t_1} \cdot \tau_1 + \overline{\varepsilon^c_2} \cdot \frac{t_2}{1-t_2} \cdot \tau_2 - \overline{\eta_n} \cdot \frac{t_1}{1-t_1} \cdot \tau(z^*), \tag{39}$$

where $\overline{\varepsilon^c_1} = \int_{z<z^*} \varepsilon^c_n z_n h(z) dz$, $\overline{\varepsilon^c_2} = \int_{z>z^*} \varepsilon^c_n z_n h(z) dz$ are income-weighted average compensated elasticities, and $\overline{\eta_n} = \int_{z=z^*} \eta_n h(z^*) dz$ is average income effect.

In contrast to the general but unpractical formulae in proposition 2, equation (39) is empirically tractable because all the terms such as the marginal tax rates, compensated elasticity and labour income distribution are observable and estimatable. In the view of sufficient statistics approach, in other words, the sufficient statistics for a small reform in the piece-wise linear labour income taxation are the income-weighted average compensated elasticities and average income effect of different groups of individuals. Moreover, it's easy to generalize equation (39) to cases of more brackets.

6 Marginal excess burden and HSV tax function

Heathcote, Storesletten and Violante (2017) show that the log-linear function $T(z) = z - (1 - \phi)/(1-p)z^{1-p}$ (hereafter HSV tax function) has a superduper approximation of the actual tax and transfer system in USA. In HSV function, p refers to the progressivity of the tax system: when $p > 0$, the tax system is progressive; conversely, when $p < 0$, the tax system is regressive; and when $p = 0$, it's a flat tax schedule. The marginal tax rates of HSV tax function are: $T'(z) = 1 - (1-\phi)z^{-p}$, indicating that the marginal tax rate for individual with labour income z increases as the progressivity of the tax system rises. Consider a tax reform $\tau(z)$ which takes on the following form:

$$\tau(z) = \frac{1-\phi}{1-p} z^{1-p} \left(\log z - \frac{1}{1-p} \right). \tag{40}$$

Then a small tax reform above raises the marginal tax rate by $\tau'(z) = (1-\phi)z^{-p} \log z$, illustrating that the change in marginal tax rate decreases as the progressivity p goes up.

Furthermore, we leave the consumption of commodities out transiently and assume the preference of individual n to be

$$u(c_n, z_n; n) = \log c_n - \left(1 + \frac{1}{\varepsilon}\right)^{-1} \left(\frac{z_n}{n}\right)^{1+\frac{1}{\varepsilon}}, \tag{41}$$

where $\varepsilon \in (0,1)$ is the compensated elasticity parameter. By solving the individual's optimization problem, we have the optimal labour income for individual n:

$$z_n = n(1-p)^{\varepsilon/(1+\varepsilon)}, \tag{42}$$

and the compensated elasticity ε_n^c as well as income effect parameter η_n[2]:

$$\varepsilon_n^c = \frac{\varepsilon}{1 + \frac{\varepsilon}{n}(1-p)^{\frac{1}{\varepsilon+1}}} > 0, \quad \eta_n = -\frac{1}{1 + \frac{1}{\varepsilon(1-p)}} < 0. \tag{43}$$

Equations (42) and (43) indicates that the progressivity p plays an important role in determining the individual's labour supply and in shaping his or her preference. When the progressivity p

[2] See the appendix for detailed derivations.

rises remarkably, the optimal labour supply z_n decreases, and the elasticity ε_n^c and income effect parameter η_n increase, implying that individual n will work less and becomes much more sensitive to the price change.

Eventually, the aggregate marginal excess burden formula in equation (35) can be simplified to:

$$AEB^L = \int_{\underline{n}}^{\overline{n}} \Big\{ -\left(1-(1-\phi)z_n^{-p}\right) \cdot \frac{1}{1+p\varepsilon_n^c}\varepsilon_n^c \cdot z_n \log z_n + \left(1-(1-\phi)z_n^{-p}\right) \cdot \frac{p\varepsilon_n^c}{1+p\varepsilon_n^c}\eta_n \cdot \frac{z_n}{1-p}\left(\log z_n - \frac{1}{1-p}\right) \Big\} f(n)dn, \quad (44)$$

where z_n satisfies the optimal solution in equation (42), and the elasticities ε_n^c and η_n satisfy the equation (43). This simplified formula allows us to discuss how a rise in the progressivity of the labour income taxation affects the efficiency cost.

7 Conclusions and future extensions

In this paper, we explore the excess burden in a mixed tax system, in which the commodity taxes are linear while the labour income taxes are non-linear. We focus on the bias between cases with and without pre-existing taxes and find the bias arises from two sources: a pure income effect of labour supply and a cross complementarity between consumption and labour supply and that the bias would disappear in a linear tax system. Besides, in a non-linear tax system, individuals' behavioral responses to tax reforms are characterized by curvature adjusted elasticities rather than standard compensated elasticities, which highlights the importance of the curvature of the non-linear tax system. Furthermore, we have deeper insights on the mutual interactions between commodity and labour supply markets via the cross complementarity $\partial x_n^{i,c}/\partial z_n$. Finally, we apply our general results of marginal excess burden to piece-wise linear and HSV tax function respectively, and obtain simplified formulae, which are empirically tractable.

Our work is still not comprehensive enough and more efforts need to be dedicated to it. First of all, an empirical and numerical analysis should be implemented immediately in order to show how large the bias is. Secondly, marginal excess burden from large tax reforms should be incorporated later because there are seldom small reforms in reality. Thirdly, apart from the marginal excess burden, is there any approach to approximate the excess burden under a non-linear tax schedule as the Harberger formula?

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Appendix A

A1 Proof of Proposition 1

Proof. Firstly, we derive the marginal excess burden from a small rise in commodity tax. Taking total derivatives of $EV(n)$ w.r.t q_j yields:

$$\frac{dEV(n)}{dq_j} = -\frac{\partial E^n}{\partial V^n} \times \left(\frac{\partial V^n}{\partial q_j} + \frac{\partial V^n}{\partial Q_n}\frac{dQ_n}{dq_j} + \frac{\partial V^n}{\partial I_n}\frac{dI_n}{dq_j} \right). \tag{A1}$$

Recall that $\partial E^n / \partial V^n \times \partial V^n / \partial I_n = 1$, and the Roy's identities, i.e. $x_n^j = -(\partial V^n / \partial q_j)/(\partial V^n / \partial I_n)$ and $z_n = -(\partial V^n / \partial Q_n)/(\partial V^n / \partial I_n)$. Then substituting equations (10) and (11) yields $\frac{dEV(n)}{dq_j} = x_n^j$. Then we calculate changes in tax revenue:

$$\begin{aligned} \frac{d}{dq_j}\left(\sum_{i=1}^{S} t_i x_n^{i,c} + T(z_n^c, \kappa) \right) &= x_n^j + \sum_{i=1}^{S} t_i \frac{dx_n^{i,c}}{dq_j} + T'(z_n)\frac{dz_n^c}{dq_j} \\ &= x_n^j + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial q_j} + \left(\sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} + T'(z_n) \right) \frac{dz_n^c}{dq_j}. \end{aligned} \tag{A2}$$

Then, sticking equation (13) and equations above yields equation (28).

Next, we derive the marginal excess burden from a small rise in labour income tax. Similarly, it's easy to obtain:

$$\frac{dEV(n)}{d\kappa} = \tau(z_n) \tag{A3}$$

$$\frac{d}{dq_j}\left(\sum_{i=1}^{S} t_i x_n^{i,c} + T(z_n^c, \kappa) \right) = \tau(z_n) + \left(\sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} + T'(z_n) \right) \frac{dz_n^c}{d\kappa} \tag{A4}$$

Then, sticking equation (22) and equations above yields equation (29). □

A2 Proof of Proposition 2

Proof. Properties such as $dEV(n)/dq_j = x_n^j$ and $dEV(n)/d\kappa = \tau(z_n)$ remain valid. Firstly, we derive $MEB^C(n)$. Let $\Delta R_1^C(n)$ and $\Delta R_1^L(n)$ denote:

$$\Delta R_1^C(n) = \sum_{i=1}^{S} t_i' x_n^{i,c}(q, v^n, z_n) - \sum_{i=1}^{S} t_i x_n^{i,c}(\hat{q}, v^n, \hat{z}_n), \tag{A5}$$

$$\Delta R_1^L(n) = T(z_n^c(q, Q_n, V^n)) - T(z_n^c(\hat{q}, \hat{Q}_n, V^n)). \tag{A6}$$

Then, taking derivatives in q_j' at $\Delta t_j = 0$, we have:

$$\frac{d\Delta R_1^C(n)}{dq_j} = x_n^j + \sum_{i=1}^{S} t_i \frac{dx_n^i}{dq_j} - \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial v^n} \cdot \left(\frac{\partial v^n}{\partial q_j} + \frac{\partial v^n}{\partial y_n}\frac{dy_n}{dq_j} + \frac{\partial v^n}{\partial z_n}\frac{dz_n}{dq_j} \right), \tag{A7}$$

$$\frac{d\Delta R_1^L(n)}{dq_j} = T'(z_n)\frac{dz_n}{dq_j} - T'(z_n)\frac{\partial z_n^c}{\partial V^n} \times \left(\frac{\partial V^n}{\partial q_j} + \frac{\partial V^n}{\partial Q_n}\frac{dQ_n}{dq_j} + \frac{\partial V^n}{\partial I_n}\frac{dI_n}{dq_j} \right). \tag{A8}$$

According to dual properties, identity $x_n^i(q, y_n, z_n) = x_n^{i,c}(q, v^n(q, y_n, z_n), z_n)$ holds true. Thus, we have $\partial x_n^{i,c}/\partial v^n \times \partial v^n/\partial y_n = \partial x_n^i/\partial y_n$. In the same way, we have $\partial z_n^c/\partial V^n \times \partial V^n/\partial I_n = \partial z_n/\partial I_n$. Besides, we substitute the Roy's identities into the above equations, and obtain:

$$\frac{d\Delta R_1^C(n)}{dq_j} = x_n^j + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial q_j} + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \cdot \frac{dz_n}{dq_j}, \tag{A9}$$

$$\frac{d\Delta R_1^L(n)}{dq_j} = T'(z_n)\frac{dz_n}{dq_j} + T'(z_n)\frac{\partial z_n}{\partial I^n} x_n^j. \tag{A10}$$

Thus we have the $MEB^C(n)$:

$$MEB^C(n) = -\sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial q_j} - \left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \frac{dz_n}{dq_j} - T'(z_n)\frac{\partial z_n}{\partial I_n} x_n^j. \tag{A11}$$

And it can be formed with sufficient elasticities:

$$\begin{aligned} MEB^C(n) = &-\sum_{i=1}^{S} t_i \varepsilon_n^{ij,c} \frac{x_n^i}{q_j} - \left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \tilde{\varepsilon}_n^{zj,c} \frac{z_n}{q_j} \\ &+ \left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \tilde{\eta}_n s_j \frac{z_n}{q_j} - T'(z_n)\eta_n s_j \frac{z_n}{q_j}. \end{aligned} \tag{A12}$$

Next, we derive $MEB^L(n)$. Let $\Delta R_2^C(n)$ and $\Delta R_2^L(n)$ denote:

$$\Delta R_2^C(n) = \sum_{i=1}^{S} t_i x_n^{i,c}(q, v^n, z_n) - \sum_{i=1}^{S} t_i x_n^{i,c}(\hat{q}, v^n, \hat{z}_n), \tag{A13}$$

$$\Delta R_2^L(n) = T(z_n^c(q, Q_n, V^n)) + \kappa\tau(z_n^c(q, Q_n, V^n)) - T(z_n^c(\hat{q}, \hat{Q}_n, V^n)). \tag{A14}$$

Then, taking derivatives in κ at $\kappa = 0$, we have:

$$\frac{d\Delta R_2^C(n)}{d\kappa} = \sum_{i=1}^{S} t_i \frac{dx_n^i}{d\kappa} - \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial v^n} \cdot \left(\frac{\partial v^n}{\partial y_n} \frac{dy_n}{d\kappa} + \frac{\partial v^n}{\partial z_n} \frac{dz_n}{d\kappa} \right), \tag{A15}$$

$$\frac{d\Delta R_2^L(n)}{d\kappa} = \tau(z_n) + T'(z_n) \frac{dz_n}{d\kappa} - T'(z_n) \frac{\partial z_n^c}{\partial V^n} \times \left(\frac{\partial V^n}{\partial Q_n} \frac{dQ_n}{d\kappa} + \frac{\partial V^n}{\partial I_n} \frac{dI_n}{d\kappa} \right). \tag{A16}$$

Then, we use Roy's identities to simplify the above equations:

$$\frac{d\Delta R_2^C(n)}{d\kappa} = \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \frac{dz_n}{d\kappa}, \tag{A17}$$

$$\frac{d\Delta R_2^L(n)}{d\kappa} = \tau(z_n) + T'(z_n) \frac{dz_n}{d\kappa} + T'(z_n) \frac{\partial z_n}{\partial I_n} \tau(z_n). \tag{A18}$$

Thus, we have $MEB^L(n)$:

$$MEB^L(n) = -\left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right) \frac{dz_n}{d\kappa} - T'(z_n) \frac{\partial z_n}{\partial I_n} \tau(z_n). \tag{A19}$$

And it can be formed with sufficient elasticities:

$$\begin{aligned} MEB^L(n) = &- \frac{\left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right)}{1 - T'(z_n)} \cdot \tilde{\varepsilon}_n^c z_n \tau'(z_n) + \frac{\left(T'(z_n) + \sum_{i=1}^{S} t_i \frac{\partial x_n^{i,c}}{\partial z_n} \right)}{1 - T'(z_n)} \cdot \tilde{\eta}_n \tau(z_n) \\ &- \frac{T'(z_n)}{1 - T'(z_n)} \cdot \eta(n) \tau(z_n). \end{aligned} \tag{A20}$$

□

A3 Derivation for equations (42) and (43)

First of all, in HSV tax function, the marginal tax rate $T'(z) = 1 - (1-\phi)z^{-p}$ and $Q_n = (1-\phi)z^{-p}$. Then, we have the first order condition:

$$-c_n \times \left(z_n^{\frac{1}{\varepsilon}} \left(\frac{1}{n}\right)^{1+\frac{1}{\varepsilon}} \right) = -Q_n = -(1-\phi)z_n^{-p}. \tag{A21}$$

Then substituting the budget constraint $c_n = Q_n z_n + I_n$ yields the optimal labour income z_n^* in equation (42).

Taking logarithm with respect to both sides of equation (A21), then we obtain:

$$\log c_n = -\frac{1}{\varepsilon}\log z_n - \frac{\varepsilon}{\varepsilon+1}\log\frac{1}{n} + \log Q_n. \tag{A22}$$

Sticking the equation above into the constraint of the expenditure minimization problem: $\log c_n - \left(1+\frac{1}{\varepsilon}\right)\left(\frac{z_n}{n}\right)^{1+\frac{1}{\varepsilon}} = \bar{u}_n$, and then taking derivatives yield:

$$\varepsilon_n^c = \frac{d\log z_n}{d\log Q_n} = \frac{1}{\frac{1}{\varepsilon} + \left(\frac{1}{n}\right)^{1+\frac{1}{\varepsilon}} z^{\frac{1}{\varepsilon}}} = \frac{\varepsilon}{1+\frac{\varepsilon}{n}(1-p)^{\frac{1}{\varepsilon+1}}}. \tag{A23}$$

As for the income effect, combine the first order condition and the budget constraint to cancel c_n out:

$$Q_n z_n + I_n = Q_n z_n^{-\frac{1}{\varepsilon}} \cdot n^{1+\frac{1}{\varepsilon}}. \tag{A24}$$

Then taking derivatives in I_n yields:

$$\eta_n = -\frac{1}{1+\frac{1}{\varepsilon(1-p)}}. \tag{A25}$$